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Assessment of one-way shear strength models for wide reinforced concrete members without stirrups in design codes

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Abstract

In the last years, many design code models of one-way shear strength have been replaced or improved. However, the Brazilian code model did not follow the same tendency. In this study, we purpose to discuss and compare the theoretical and mechanical basis of different design code models for one-way shear of reinforced concrete members without stirrups. To this, we bring together the models provided in design codes for concrete structures from different countries. Apart from that, we compare the experimental shear capacities with the one provided by these models for two tests described in the literature. The ratio between experimental shear capacities varied between 0.80 to 1.33 for the test W105 and between 0.43 to 0.91 for the test AT-1. The main parameter changed between these two tests was the member thickness, which changed from 0.21m in the first to 0.91 m in the second. In this study, we conclude that design code models that incorporate parameters to deal with the size effect and shear slenderness show a higher level of accuracy regardless of the member thickness. Furthermore, the model of shear strength provided by the ABNT NBR 6118:2014 needs to be replaced or improved to be used for members of large thicknesses.

Keywords

shear strength; reinforced concrete members; members without stirrups.

1 Introduction

The shear strength of reinforced concrete members without stirrups is one of the main topics studied in the literature. Proof of that is a large number of semi-empirical and mechanical based models described in the technical literature and provided by design codes (Bentz et al., 2006; Cladera et al., 2016; Muttoni & Fernandez Ruiz, 2008). Between the reasons for this, we can cite the absence of well-accepted models of shear strength that balance accuracy with the easiness of use for the daily engineer. Apart from that, many researchers disagree about the main shear transfer mechanisms and on what drives the shear failure.

In the last ten years, several professional associations replaced or improved models of shear strength in design codes for one-way reinforced concrete members without stirrups (AASHTO, 2017; ACI Committee 318, 2019; AS 3600, 2018; fib, 2012). This move occurs mainly to correct unsafe predictions of shear strength for members with large thicknesses. Since shear failure usually occurs without reinforcement yielding, they can develop sudden and require costly repair tasks (Burgoyne & Scantlebury, 2006; Johnson et al., 2007; Kottari et al., 2020). On the contrary way to most countries, the one-way shear model provided in the Brazilian code (ABNT NBR 6118 2014) for reinforced concrete members without stirrups was not replaced or improved in the last revision. Therefore, unsafe predictions of shear strength can occur if the model is used outside of the conditions recommended in the technical literature (Joint Technical Committee ABECE/IBRACON, 2015).

We purpose to discuss the limitations of the Brazilian code model for one-way shear strength of RC members without stirrups and review the main models of shear strength used in codes of practice. For this, we bring together the main shear models provided by different design codes and we explain the background of each model. Apart from that, and we compared the experimental and predicted shear capacities by different models for two tests: (i) test W105 from Conforti et al. (Conforti, Minelli, and Plizzari 2017) and (ii) test A1-T1 from Lubell (Lubell 2006).

2 One-way shear models

2.1 Brazilian code - ABNT NBR 6118:2014

According to item 19.4.1 of the Brazilian code, the one-way shear strength of reinforced concrete slabs without transverse reinforcement is calculated as follow:

$$V_{Rd,c,NBR} = \left[\tau_{Rd} \cdot k\left(1,2 + 40 \cdot \rho_1\right) + 0,15 \cdot \sigma_{cp}\right] \cdot b_w \cdot d \tag{1}$$

$$\tau_{Rd} = 0,25 \cdot f_{ctd} \tag{2}$$

$$f_{ctd} = f_{ctk,\inf} / \gamma_c \tag{3}$$

$$f_{ctk,\inf} = 0,7 \cdot f_{ctm} \tag{4}$$

$$f_{ctm} = \begin{cases} 0,3 \cdot \sqrt[3]{f_{ck}^{\;2}} & \text{for } f_{ck} \le 50MPa \\ f_{ctm} = 2,12 \cdot \ln(1 + 0,11 \cdot f_{ck}) & \text{for } 50\ MPa < f_{ck} \le 90MPa \end{cases} \tag{5}$$

$$\rho_1 = \frac{A_{s1}}{b_w \cdot d} \le \left|0,02\right| \tag{6}$$

$$\sigma_{cp} = N_{Sd} / A_c \tag{7}$$

τ_{Rd} is the shear strength stress calculated from the concrete's tensile strength; k is a coefficient that depends on the longitudinal reinforcement ratio that reaches the supports and the effective depth d, which works like a simplified size effect parameter. ρ_l is the longitudinal reinforcement ratio; σ_{cp} is the normal stress due to prestress, and b_w and d are the cross-section dimensions in a distance d from the support edge.

2.2 European code - NEN 1992-1-1:2005

According to §6.2.2 of the European code NEN 1991-1:2005 (CEN, 2005), the one-shear strength of reinforced concrete members without shear reinforcement is calculated by (SI units: f_{ck} in [MPa]):

$$V_{Rd,c,EC2} = \left[C_{Rd,c} k (100 \rho_l f_{ck})^{1/3} + k_1 \sigma_{cp}\right] b_w d \ge \left(v_{\min} + k_1 \sigma_{cp}\right) b_w d \tag{8}$$

$C_{Rd,c}$ is an empirical factor used for characteristic shear strength calculations and it was derived from comparison with experimental results (Regan, 1987) and calibrated through reliability analysis on 176 beams tests (Konig & Fischer, 1995). $C_{Rd,c}$ may be calculated as $0.18/\gamma_c$; where γ_c is the partial safety factor for concrete in shear, generally equal to 1.5; k and v_{min} are coefficients that assume values of according with the national annex of each country, with recommended values in the Dutch annex (CEN, 2005) of $k = 0.15$ and $v_{min} = 0.035 k^{3/2} \sqrt{f_{ck}}$. The effect of loads applied within $0.5d \le a_v \le 2d$ to the shear force V_{Ed} may be multiplied by $0.25 \le \beta_{EC} \le 1$, which accounts for the improved arching action for this load.

While some models allow deal with the problem of members under concentrated loads close to the support by reductions of the acting shear load by the sake of simplicity (CEN, 2005; fib, 2012). Others consider that for non-slender members (a/d<2.5), the problem should be solved by strut and tie-methods (Jackson et al., 2007; Vollum & Fang, 2015).

2.3 American code - ACI 318:2014

The model from ACI 318:2014 was developed by MacGregor & Hanson (MacGregor and Hanson 1969) and present two approaches for calculation. The simplified method considerer only the influence of the concrete tensile. Note that this method does not considered the influence of the longitudinal reinforcement ration, shear slenderness, and size effect in the shear strength. In the detailed formula, on the other hand, it is accounted for the influence of the shear slenderness and longitudinal reinforcement ration in the shear strength. However, for many years the first equation was well accepted for engineers due to its simple format for the daily practice and the conservative results for design. Apart from that, the detailed equation also does not consider the size effect, which was the most concerning for many years until the replacement of this code in 2019.

$$V_{ACI,2014,simplified} = 0.17\sqrt{f_{ck}}b_w d_l \tag{9}$$

$$V_{ACI,2014,\text{detailed}} = \left(0.16\sqrt{f_{ck}} + 17\rho_l \frac{Vd_l}{M}\right)b_w d_l \leq 0.29\sqrt{f_{ck}}b_w d_l \tag{10}$$

$$\text{limits:}\begin{cases} \{V_{ACI,2014} \leq 0.29\sqrt{f_{ck}}b_w d \\ \dfrac{Vd_l}{M} \leq 1 \end{cases} \tag{11}$$

2.4 fib Model Code 2010

The fib Model Code 2010 (fib, 2012) shear provisions introduce the approach entitled "Levels of Approximation - LoA" for shear calculations in the design and assessment of existing structures. This approach means that different levels of accuracy in the prediction of the shear strength may be required according to the design situation, which will demand less or more refined calculations models. For instance, in LoA I, fewest parameters are required for the sake of simplicity and conservative assumptions are made in order to reach safe predictions of capacity, which is the preferable case for the design of new structures or preliminary calculations of more complex structures. In LoA II of approximation, a higher number of parameters are required, which will increase the calculations but will improve the accuracy of the prediction, with a lower level of conservatism. Both methods are raised in the Simplified Modified Compression Field Theory (SMCFT), which warrants a physical explanation for most parameters (Bentz et al., 2006).

The shear strength of reinforced concrete members without shear reinforcement is calculated in §7.3.3.2 of fib Model Code 2010 by (with f_{ck} in [MPa]):

$$V_{MC2010} = k_v \frac{\sqrt{f_{ck}}}{\gamma_c} z b_w \quad (\text{with } \sqrt{\mathrm{f}_{ck}} \leq 8\text{MPa}) \tag{12}$$

In high strength concretes, shear cracks go through the aggregate particles rather than around, which provides smoother surfaces of contact between the two portions of the cracks and decrease the aggregate interlock (Walraven, 1981). Due to this behavior, the value $\sqrt{f_{ck}}$ is limited to a maximum of 8 MPa. k_v is the factor accounting the tensile stresses in the cracked concrete by a combination of a strain effect factor (ε_x) and size effect factor (member size z), and is equal to the factor β in the original Simplified Modified Compression Field Theory (Bentz et al., 2006).

$$k_v(II) = \frac{0.4}{1+1500\varepsilon_x}\frac{1300}{1000+k_{dg}z} \tag{13}$$

The strain effect accounts for the reduced capacity of transfer shear forces under large crack openings, which means considerer the shear slenderness effect. The aggregate size is considered within k_{dg}, where d_g is the specified maximum size of the aggregate (Sigrist et al., 2013). Due to the same reason of limit $\sqrt{f_{ck}} \leq 8MPa$, d_g should assume a value zero for high strength concretes. k_{dg} is defined as:

$$k_{dg} = \frac{32}{16+d_g} \geq 1.75 \tag{14}$$

In the LoA I, k_v is simplified by assuming k_{dg} = 1.25, which means a maximum aggregate size > 9.6 mm, and ε_x = 0.00125 or half of the yield strain of the longitudinal reinforcing bars with f_{yk} = 500 MPa (Sigrist et al., 2013). Therefore, k_v for LoA I becomes:

$$k_v(I) = \frac{180}{1000+1.25z} \tag{15}$$

Similar shear provisions are included in the Canadian design code (CSA Committee A23.3, 2004).

2.5 American code - ACI 318:2019

In the ACI 318:2019 shear provisions, the shear capacity owing to concrete is calculated in different ways for members with or without stirrups, as follows in S.I. units:

If $A_v \geq A_{v,min}$, Either of:

$$V_c = \left[0.17\lambda\sqrt{f_c} + \frac{N_{Ed}}{6\cdot A_g}\right]b_w d \tag{16}$$

$$V_c = \left[0.66\lambda(\rho)^{1/3}\sqrt{f_c} + \frac{N_{Ed}}{6A_g}\right]b_w d \tag{17}$$

If $A_v < A_{v,min}$

$$V_c = \left[0.66\lambda_s\lambda(\rho)^{1/3}\sqrt{f_c} + \frac{\sigma_{cp}}{6A_g}\right]b_w d \tag{18}$$

$$\lambda = \begin{cases} 1, \text{ to normalweight aggregate} \\ 0.75, \text{ to lightweight aggregate} \end{cases} \tag{19}$$

$$\lambda_s = \sqrt{\frac{2}{1+0.004d}} \leq 1, \text{ with } d \text{ in [mm] and } f_c \text{ in [MPa]} \tag{20}$$

$$\text{limits}: \begin{cases} \sqrt{f_c} \leq 8.3MPa \\ V_{ACI} \leq 0.42\lambda\sqrt{f_c}b_w d \\ \dfrac{N_{Ed}}{A_g} \leq 3.45MPa \end{cases} \tag{21}$$

A_v is the shear reinforcement ratio; λ is the coefficient to account for reduced shear capacities with lightweight concretes. The new provisions from ACI 318:2019 (ACI Committee 318, 2019) outstanding compared to the previous one (ACI Committee 318, 2014) by including a size effect parameter λ_s, which is based on the fracture mechanics (Bažant et al., 2007). However, the new model also provides improvements regarding its physical meaning:

- While in the previous version, the ultimate shear strength was assumed as the diagonal cracking strength (Belarbi et al., 2017), which is not based in a physical model, the updated version agrees better with models based on the compression chord capacity.
- The influence of axial loads is better described in the new model, since the effect of axial loads and prestressing forces are dealt with the same formulation (Kuchma et al., 2019), while in the previous version there were different formulas to take into account the same effect.

Different formulas calculate the concrete contribution to the shear strength for members with and without shear reinforcement. Note that for members with the minimum amount of shear reinforcement the size effect is not considered.

2.6 Swiss Code SIA 262:2013

The shear strength calculation model provided in the Swiss code SIA 262: 2013 is based on the critical shear crack theory (CSCT) (Muttoni & Fernandez Ruiz, 2008). The shear strength per unit length is given by:

$$v_{Rd,c,SIA} = k_d \cdot \tau_{cd} \cdot d \tag{22}$$

With:

$$k_d = \frac{1}{1+\varepsilon_v \cdot d \cdot k_{g,SIA}} \tag{23}$$

$$\tau_{cd} = \frac{0,3 \cdot \sqrt{f_{ck}}}{\gamma_c} \quad \text{with } f_{ck} \text{ in [MPA]} \tag{24}$$

$$k_{g,SIA} = \frac{48}{16+d_g} \quad \text{for nomal weigth concrentes and } f_{ck} < 70MPa \tag{25}$$

If the reinforcement remains in the elastic regime, the longitudinal strain in the design section ε_v shall be determined according to:

$$\varepsilon_v = \frac{f_{yd}}{E_s} \cdot \frac{m_{Ed}}{m_{Rd}} \tag{26}$$

If the plastic deformations of the reinforcement cannot be neglected ε_v can be calculated by:

$$\varepsilon_v = 1,5 \cdot \frac{f_{yd}}{E_s} \tag{27}$$

The acting bending moment per unit length m_{Ed} and the yielding moment per unit length m_R shall be calculated at a control section located at a distance *d/2* from the plate of load introduction or the edge of the support. The flexural strength m_R can be calculated, assuming a plastic behaviour of the reinforcement after yielding, as:

$$m_{Rd} = \rho \cdot d^2 \cdot f_{yd} \cdot \left(1 - \frac{\rho \cdot f_{yd}}{2 \cdot f_{cd}}\right) \tag{28}$$

3 Reference tests

In this study, we used as reference tests: (i) W105-phi14 from Conforti et al. (Conforti et al. 2017) and (ii) test A1T1-East from Lubell (Lubell 2006). Table 1 shows an overview of the section and material parameters of these tests. Table 1 shows that the main geometrical difference between these two tests refers to the member thicknesses *d*. Since both tests were loaded over the full width, we neglect the influence of the member's width in the shear behavior. In Table 1, the ratio *a/d* is the shear slenderness parameter used in this study, on which "*a*" is the distance between axes of support and loading plate.

Table 1 - Parameters overview from the reference tests.

Parameter	Test 1	Test 2
Test	W105Ø14	A1T1
b (m)	0.105	2.016
d (m)	0.21	0.916
a/d [-]	2.50	2.95
ρ_l (%)	1.4	0.76
ρ_t (%)	0	0.018
f_y (MPa)	546	465
f_{cm} (MPa)	44.4	64
d_{ag} (mm)	16	10
V_{exp} (kN)	26	1133

4 Results and Discussions

In this study, we compared experimental and predicted shear capacities by different design code models: (ABNT NBR 6118 2014); EN 1992-1-1:2005 (CEN, 2005); ACI 318: 2014 in the simplified and detailed version (ACI Committee 318, 2014); ACI 318:2019 (ACI Committee 318, 2019), Model Code 2010 with Levels of Approximation I and II (fib, 2012) and SIA 262:2013.

Figure 1 show that the ratio between experimental and predicted shear capacities V_{exp}/V_{cal} varied between 0.75 and 1.85 for the test W105Φ14 of Conforti et al. (Conforti et al. 2017). Both limits refer to the predictions with different levels of approximation with the Model Code 2010 formulation, being the most conservative prediction provided by the LoA I. Despite the LoA II provided an unsafe prediction of shear strength, the calculated shear strength overestimate the experimental one in less than 25%.

The current American model (ACI 318:2019) showed an conservative prediction of the shear strength. On the other hand, the Brazilian model (ABNT NBR 6118:2014) show a unconservative prediction with the ratio V_{exp}/V_{cal} = 0.80. The prediction of the Swiss Code was almost equal to the one from ABNT NBR 6118:2014 for this test. The best accuracy was reached with the detailed calculation of the ACI 318:2014, with a ratio V_{exp}/V_{cal} equal to 1.00. This model includes the shear slenderness effect by the ratio *M/Vd*, which improves the accuracy in this case compared to the simplified version of ACI 318:2014.

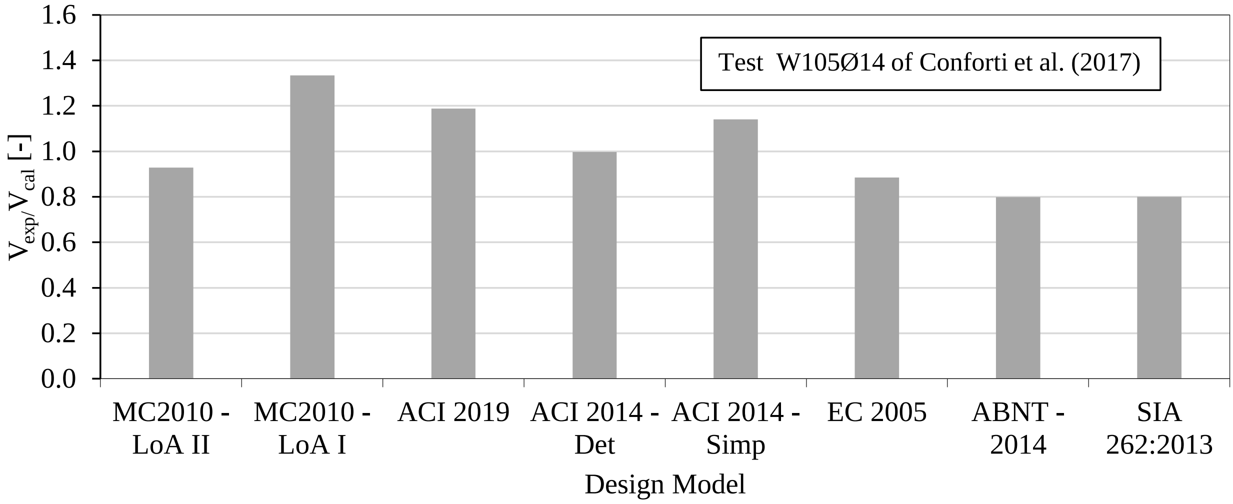

Figure 1 - Comparison between experimental and predicted shear capacities with shear provision from different design codes for the test W105Φ14 of Conforti et al. (Conforti et al. 2017)**.**

Figure 2 shows the comparison between experimental and predicted shear capacities with the same models, but for the test AT-1-East of Lubell (Lubell 2006), with a member thickness close to 1.00 m. In this analysis, the ratio V_{exp}/V_{cal} varied between 0.46 and 0.91. Note that all models predicted higher shear capacities than the verified one, hence unsafe predictions. The best prediction, V_{exp}/V_{cal} closer to 1, was reached with the LoA II of the Model Code 2010, whose prediction overestimated the experimental shear capacities in less than 10%. The best result is explained by the physical derivation of this model, which accounts for the strain effect (shear slenderness) and member size effect (size effect).

The most unsafe predictions occurred with the models provided in the ABNT NBR 6118:2014 and ACI 318:2014, with ratios V_{exp}/V_{cal} of 0.52 and 0.44. Note that in these cases the shear models overestimated the shear capacities in more than 40%, which is a critical result. Due to critical results like this, the American code included in the last revision a size effect parameter. The shear strength with the ACI 318:2019 overestimates the shear capacity in less than 15%. Therefore, significative improvement in the shear strength model of the ACI 318:2019 was reached regarding the size effect. In Figure 2, the level of accuracy reached by the SIA 262:2013 was comparable to the one from Model Code 2010 for LoA II. A similar level of approximation for these models indicates that they deal well with the strain effect (shear slenderness) and size effect.

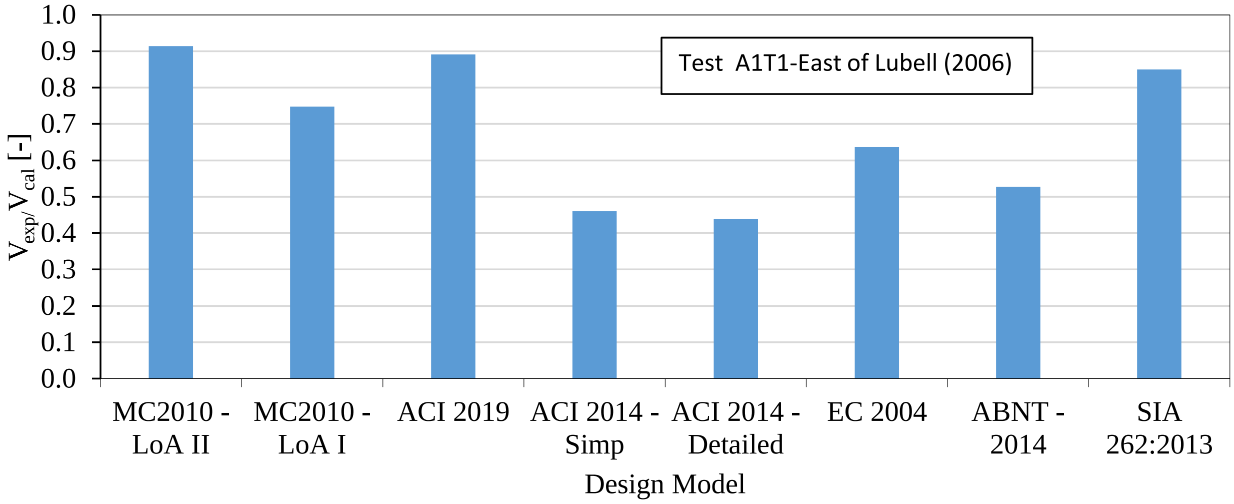

Figure 2 - Comparison between experimental and predicted shear capacities with shear provision from different design codes for the test A1T1-East of Lubell (Lubell 2006)**.**

5 Conclusions

- Mechanical models of shear strength have been adopted in most codes of design of concrete structures, except the Brazilian and European. However, the European code model is currently being revised and shall be replaced by a mechanical based model also. Despite the American code model does not deal directly with the shear slenderness effect, it was included as an improved size effect parameter, which can be considered a step forward compared to the Brazilian code.
- Semi-empirical models as provided in the ACI 318:2014 and ABNT NBR 6118:2014 may overestimate the shear capacity in more than 40% for members with large thickness. In the ACI 318:2019, a size effect parameter improved the provisions for large thickness members significantly. In the Brazilian model, the improvement or replacement of the size effect parameter is needed.

Mechanical models of shear strength, such as the provided in fib Model Code 2010 (fib,2012) cover well both applications in new design and assessment of existing structures since it includes different formulas according to the required level of approximation and deals physically with the size effect and the shear slenderness influence.

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References

AASHTO. (2017). *AASHTO LRFD Bridge Design Specifications* (8th Editio). American Association of State Highway and Transportation Officials (AASHTO).

ABNT NBR 6118. (2014). *NBR 6118: Design of concrete structures — Procedure (in portuguese).* Associação Brasileira de Normas Técnicas (ABNT).

ACI Committee 318. (2014). *Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary* (Vol. 11, p. 6858). American Concrete Institute, Farmington Hills, MI.

ACI Committee 318. (2019). *Building Code Requirements for Structural Concrete (ACI 318-19)* (p. 988). American Concrete Institute, Farmington Hills, MI.
AS 3600. (2018). *Concrete structures* (Fifth edit). SAI Global Limited under licence from Standards Australia Limited.
Bažant, Z. P., Yu, Q., Gerstle, W., Hanson, J., & Ju, J. W. (2007). Justification of ACI 446 Proposal for Updating ACI Code Provisions for Shear Design of Reinforced Concrete Beams. *ACI Structural Journal*, *104*(5), 601–610. https://doi.org/10.14359/18862
Belarbi, Bentz, Cladera, Frosch, Li, Park, H.-G., & Reineck. (2017). Proposals for New One-Way Shear for the 318 Building Code. *Concrete International*, *39*(9), 29–70.
Bentz, E. C., Vecchio, F. J., & Collins, M. P. (2006). Simplified modified compression field theory for calculating shear strength of reinforced concrete elements. *ACI Structural Journal*, *103*(4), 614–624. https://doi.org/10.14359/16438
Burgoyne, C., & Scantlebury, R. (2006). Why did Palau Bridge collapse? *Structural Engineer*, *84*(11), 30–37.
CEN. (2005). *EN 1992-1-1: Eurocode 2: Design of concrete structures -Part 1-1: General rules and rules for buildings, EN 1992-1-1:2005*. Comité Européen de Normalisation, Brussels, Belgium.
Cladera, A., Marí, A., Bairán, J. M., Ribas, C., Oller, E., & Duarte, N. (2016). The compression chord capacity model for the shear design and assessment of reinforced and prestressed concrete beams. *Structural Concrete*, *17*(6), 1017–1032. https://doi.org/10.1002/suco.201500214
Conforti, A., Minelli, F., & Plizzari, G. A. (2017). Influence of width-to-effective depth ratio on shear strength of reinforced concrete elements without web reinforcement. *ACI Structural Journal*, *114*(4), 995–1006. https://doi.org/10.14359/51689681
CSA Committee A23.3. (2004). *Design of concrete structures (CSA A23.3-04)*. Mississauga: Canadian Standards Association.
fib. (2012). *fib Model Code for Concrete Structures 2010: Vol. Vol. 1-2*. Ernst & Sohn - fédération internationale du béton, Bulletin 65, Lausanne, Switzerland.
Jackson, P., Salim, S., Shave, J., & Denton, S. (2007). Shear enhancement at short shear spans in en 1992. *Structural Engineer*, *85*(23–24), 37–42.
Johnson, P. M., Couture, A., & Nicolet, R. (2007). *Report of the Commission of Inquiry into the Collapse of a Portion of the de la Concorde Overpass*.
Joint Technical Committee ABECE/IBRACON. (2015). *ABNT NBR 6118:2014 - Comentários e Exemplos de Aplicação* (1º Ed.). IBRACON.
Konig, G., & Fischer, J. (1995). Model uncertainties concerning design equations for the shear capacity of concrete members without shear reinforcement. In CEB (Ed.), *In CEB Bulletin 224, Model Uncertainties and Concrete Barrier for Environmental Protection* (pp. 49–100).
Kottari, A., Shing, P. B., & Bromenschenkel, R. (2020). Shear Behavior of Exterior Non-Isolated Shear Keys in Bridge Abutments. *ACI Structural Journal*, *117*(2), 225–238. https://doi.org/10.14359/51721317
Kuchma, D. A., Wei, S., Sanders, D. H., Belarbi, A., & Novak, L. C. (2019). Development of One-Way Shear Design Provisions of ACI 318-19 for Reinforced Concrete. *ACI Structural Journal*, *116*(4), 285–295. https://doi.org/10.14359/51716739
Lubell, A. S. (2006). *Shear in Wide Reinforced Concrete Members*. PhD dissertation. University of Toronto, Canadá.
MacGregor, J. G., & Hanson, J. M. (1969). Proposed Changes in Shear Provisions for Reinforced and Prestressed Concrete Beams*. *ACI Journal Proceedings*, *66*(4), 276–288. https://doi.org/10.14359/7360
Muttoni, A., & Fernandez Ruiz, M. (2008). Shear Strength of Members without Transverse Reinforcement as Function of Critical Shear Crack Width. *ACI Structural Journal*, *105*(2), 163–172. https://doi.org/10.14359/19731

Regan, P. E. (1987). *Shear Resistance of Members without Shear Reinforcement; Proposal for CEB Model Code MC90* (pp. 1–28).

Sigrist, V., Bentz, E., Ruiz, M. F., Foster, S., & Muttoni, A. (2013). Background to the *fib* Model Code 2010 shear provisions - part I: beams and slabs. *Structural Concrete*, *14*(3), 195–203. https://doi.org/10.1002/suco.201200066

Vollum, R. L., & Fang, L. (2015). Shear enhancement near supports in RC beams. *Magazine of Concrete Research*, *67*(9), 443–458. https://doi.org/10.1680/macr.14.00309

Walraven, J. C. (1981). Fundamental Analysis of Aggregate Interlock. *Journal of the Structural Division, ASCE*, *107*(11), 2245–2270.